RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

FIRST YEAR [2018-21]

B.A./B.Sc. FIRST SEMESTER (July – December) 2018

Mid-Semester Examination, September 2018

Date : 24/09/2018

MATHEMATICS (Honours)

Time : 11 am – 1pm

Paper: I

Full Marks : 50

[Use a separate Answer Book for each group]

GROUP – A

Module 1

Answer **any three** from question nos. 1 to 5 : (3 × 4)

1. For any finite set A, if $f : A \to A$ is injective, then show that f is surjective.
2. Let $G = \left\{ \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix} \middle| n \in \mathbb{Z} \right\}$. Show that G becomes a group under usual matrix multiplication.
3. Show that a group $(G,*)$ is commutative if and only if $(a*b)^2 = a^2 * b^2 \forall a,b \in G$.
4. Show that the number of even permutations in S_n is same as that of the odd permutations.
5. If $\beta = (1\ 2\ 3)(1\ 4\ 5) \in S_{1952}$ then write β^{99} in cycle notation.

Answer **any one** from question nos. 6 & 7: (1 × 7)

6. Prove that the set of all algebraic numbers is countable.
7. Let $S \subseteq \mathbb{R}$ be a non-empty bounded set. Let $S_1 = \{|x-y| \mid x, y \in S\}$.

   Find the supremum and infimum of S_1.

Answer **any one** of question from 8 & 9. (1 × 6)

8. a) Prove using mathematical induction that $1 + \frac{1}{4} + \cdots + \frac{1}{n^2} \le 2 - \frac{1}{n}, \ \forall\ n \in \mathbb{N}$.

   b) Prove that $\sqrt[3]{3}$ is an irrational number. (3 + 3)
9. a) Let $x \in \mathbb{R}$. Prove that there exists an integer $n \in \mathbb{Z}$ such that $n \le x \le n+1$.

   b) Let A, B be two infinite sets. Show that $A \times B$ is countable if and only if $A \cup B$ is countable. (3 + 3)

GROUP – B

Module 2

10. Answer **any two** questions of the following**:** (2 × 5)

    a) Solve the equation $(2x^2y - 3y^4)dx + (3x^3 + 2xy^3)dy = 0$.

b) Solve, using the method of variation of parameters, the equation $\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2$.

c) Reduce the differential equation $y=2px-p^2y$ to Clairaut's form by the substitutions $y^2=Y, \mathrm{x}=\mathrm{X}$ and then obtain the complete primitive and singular solution, if any.

11. Answer **any one** question of the following: (1 × 3)

a) Solve the equation $p^2+p-6=0, \left(p=\frac{dy}{dx}\right)$

b) Solve : $x\frac{dy}{dx}-y=x\sqrt{x^2+y^2}$

Answer **any one** from question nos.12 & 13: (1 × 6)

12. Show that the condition that the triangle formed by the lines $ax^2+2hxy+by^2=0$ and $lx+my=1$ is right angled if $(a+b)(al^2+2hlm+bm^2)=0$.

13. If $ax^2+2hxy+by^2+2gx+2fy+c=0$ represents a pair of intersecting straight lines, show that the area formed by the bisectors of the angles between them and the x-axis is $\frac{1}{2}\cdot\frac{g^2-ca}{h^2-ab}\cdot\frac{\sqrt{(a-b)^2+4h^2}}{|h|}$

Answer **any two** from questions nos. 14, 15 & 16: (2 × 3)

14. Show that the points (2,4,6), (3,4,5), (4,4,4) and (5,4,3) are coplanar.

15. Let $\vec{a},\vec{b},\vec{c}$ be respectively the position vectors of the vertices *A*, *B*, *C* of the ΔABC. Show that the position vector of its orthocentre is $\frac{\tan A\vec{a}+\tan B\vec{b}+\tan C\vec{c}}{\tan A+\tan B+\tan C}$.

16. Let *ABCD* be a quadrilateral and *P*, *Q* be respectively the midpoints of the diagonals *AC* and *BD*. Show that $AB^2+BC^2+CD^2+DA^2=AC^2+BD^2+4PQ^2$.

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